WALL CONDUCTION IN A HIGHLY MAGNETIZED PLASMA

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The electron current along the electric field might appear to tend to zero for $\omega_e\tau_e \to \infty$ in a low-density completely ionized plasma in mutually perpendicular electric and magnetic fields; but this current is not small in all known real systems, and, in any case, it is larger by many orders of magnitude than that calculated from classical formulas. This anomalous conductivity is usually ascribed to noise in the plasma.

However, there is a class of plasma systems for which the anomalous conductivity can be explained, at least partially, in a different way; in this class we should include homopolar systems and also certain accelerators. These systems have insulated walls met almost at right angles by the magnetic field lines, and the electric field lies almost exactly along the wall.

If the plasma density is so small that the electron mean free path is much greater than the transverse size of the channel (along the magnetic field), the conductivity of the discharge gap should be affected by the collisions of the electrons with the wall. This effect tends to be of regular character, in contrast to noise; it should appear in some form even when the magnetic field is zero, but the effects of collisions of electrons with the wall become clearer when the magnetic field is strong, since the ordinary conductivity of the plasma is largely suppressed.

The conductivity due to wall collision for $\omega_e\tau_e \to \infty$ will be called wall conductivity, since the current flows in a thin layer near the wall.

The following is the physical reason why the conductivity is affected by collision of electrons with the walls.

An electron drifting in crosses (mutually perpendicular) fields **E** and **H** acquires a velocity component transverse to these of w = cE/H. This velocity is lost on collision with the wall, and so the electron is displaced by a distance on the order of the cycloid height.

The drift builds up in a layer whose thickness is of the following order:

$$\sim \rho_T \equiv \frac{c_{Te}}{\omega_e}, \tag{1}$$

and hence the current flows along **E** within a layer of this thickness. Here c_{Te} is the characteristic thermal velocity of the electrons, ρ_T is the Larmor radius of an electron calculated from this velocity, and ω_e is the Larmor frequency.

This localization in a thin layer allows one, in principle, to distinguish the anomalous conductivity due to noise from the wall conductivity.

There must be a substantial change in electron speed on collision with the wall in order to produce this electron displacement. This can occur either by direct interaction with the surface or by reflection from the Delye layer. The latter is the more common in discharges. The wall conductivity can be varied within certain limits via the roughness of the wall.

If the thickness of the reflecting layer in a discharge is much less than ρ_T, reflection in general will be accompanied by change in the adiabatic invariant $v_\perp^2/H$, and hence by heating or cooling of the electron component. The invariant will not change if the thickness of the reflecting layer is much greater than ρ_T. Then T_e does not vary and, if the conditions are not too unusual,* there will be no electron drift along the **E** field.

To calculate the wall conductivity we need, in principle, to know the following function**:

$$S = S(v_x', v_y', v_z'; v_x, v_y, v_z \mid x, z), \tag{2}$$

which defines the probability of occurrence of a reflected particle with the velocity $\mathbf{v}' = (v_x', v_y', v_z')$ when an electron with velocity $\mathbf{v} = (v_x, v_y, v_z)$ strikes a point P = (x, 0, z). Unfortunately, S is determined not only by the physical properties of the insulator but also by those of the adjacent plasma, so there are major difficulties in calculating S and in deriving it by experiment.

From $S(\mathbf{v}', \mathbf{v}; P)$ and the distribution function $f_+(\mathbf{v}, x, 0, z)$ for the incident particles we find the distribution function for the reflected particles:

$$f_-(\mathbf{v}', x, 0, z) = \int d\mathbf{v} S(\mathbf{v}', \mathbf{v} \mid x, z) f_+(\bar{\mathbf{v}}, x, 0, z). \tag{3}$$

Here we envisage the limiting case $\omega_e\tau_e \to \infty$, so the motion of a particle within the body of the plasma should be described via Vlasov's equations with self-consistent **E** and **H**.

It is extremely difficult to consider the processes as a whole in such a system while incorporating (3), so we consider here a model that provides a clear illustration of the effects.

Example. We assume that, near some point on the wall, the magnetic field is homogeneous and perpen-

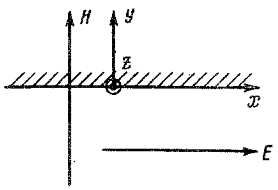

Fig. 1

*In particular, if there are no ridges large relative to ρ_T on the insulator.

**We use a coordinate system in which the y-axis is perpendicular to the wall.

dicular to the wall, while the electric field is parallel to the latter (Fig. 1). We also assume that the reflecting layer is much less than ρ_T in thickness and that its thickness can be neglected in the calculation.

Then the incident and reflected particles move trochoidally:

$$x = a - (A / \omega_e)\cos(\omega_e t + \alpha),$$
$$v_x = A \sin(\omega_e t + \alpha),$$
$$y = v_y t,\quad v_y = v_{y_0} = \text{const},\qquad w = cE / H,$$
$$z = b + wt + (A / \omega_e)\sin(\omega_e t + \alpha),$$
$$v_z = w + A\cos(\omega_e t + \alpha). \tag{4}$$

The time should be eliminated from the integrals of (4) via $y = v_y t$ in considering static problems.

If we know $f_+(v, x, 0, z)$ and $f_-(v, x, 0, z)$ at the surface of the wall, we can use (4) to find the values of these functions for any y by substituting for v, x, and z the quantities

$$v_x \to v_x \cos\omega_e \frac{y}{v_y} - (v_z - w)\sin\omega_e \frac{y}{v_y},\qquad v_y \to v_y,$$
$$v_z \to w + (v_z - w)\cos\omega_e \frac{y}{v_y} + v_x \sin\omega_e \frac{y}{v_y},$$
$$x \to x + \frac{v_z - w}{\omega_e} - \frac{1}{\omega_e}\left[v_x \sin\omega_e \frac{y}{v_y} + (v_z - w)\cos\omega_e \frac{y}{v_y}\right]$$
$$z \to z - w\frac{y}{v_y} + \frac{v_x}{\omega_e} -$$
$$- \frac{1}{\omega_e}\left[v_x \cos\omega_e \frac{y}{v_y} - (v_z - w)\sin\omega_e \frac{y}{v_y}\right]. \tag{5}$$

For the current distribution in the plasma

$$\mathbf{j} = -e\int \mathbf{v}(f_+ + f_-)\,d\mathbf{v}. \tag{6}$$

We consider the calculation scheme via a simple example in which f_+ is Maxwellian with a displacement in velocity space equal to w:

$$f_+ = \frac{1}{N_+}\exp\left\{-\frac{1}{c_{T0}^2}\left[v_x^2 + v_y^2 + (v_z - w)^2\right]\right\}. \tag{7}$$

We take f_- as being an undisplaced Maxwellian function:

$$f_- = \frac{1}{N_-}\exp\left\{-\frac{1}{c_{T1}^2}\left[v_x^2 + v_y^2 + v_z^2\right]\right\},$$
$$c_T^2 = \frac{kT_e}{m} \tag{8}$$

and the normalization factors are

$$N_+ = \frac{2n_+}{\pi^{3/2} c_{T0}^3},\quad N_- = \frac{2n_-}{\pi^{3/2} c_{T1}^3}, \tag{9}$$

in which n_+ and n_- are the densities of the particles incident on and reflected by the wall, respectively. If there is no loss of particles,

$$n_+ c_{T0} = n_- c_{T1}. \tag{10}$$

Substitution of (5) into (7) and (8) readily shows that (7) does not alter, while (8) becomes

$$f_- = \frac{1}{N_-}\exp\left\{-\frac{1}{c_{T1}^2}\left[v_x^2 + v_y^2 + (v_z - w)^2 + w^2 + 2w(v_z - w)\cos\omega_e \frac{y}{v_y} + 2wv_x \sin\omega_e \frac{y}{v_y}\right]\right\}. \tag{11}$$

We are interested in the x-component of the current, and so we calculate only that component.

Since $f_+(V_x)$ is even, we have

$$j_x = -e \iiint\limits_{v_y=-\infty}^{0} v_x f_- d\mathbf{v}. \tag{12}$$

Substitution from (9) and (11) gives

$$j_x = e\frac{2n_+}{\sqrt{\pi}}\frac{c_{T0}}{c_{T1}} c\frac{E}{H}\int_{-\infty}^{0}\sin\frac{y\omega_e}{c_{T1}\alpha}e^{-\alpha^2}d\alpha. \tag{13}$$

Then the distribution of the current density near the wall is defined by

$$Y(k) \equiv \int_{-\infty}^{0}\sin\frac{k}{\alpha}e^{-\alpha^2}d\alpha,\quad k \equiv \frac{y\omega_e}{c_{T1}} \equiv \frac{y}{\rho_T}. \tag{14}$$

The form of Y(k) can be deduced from an equation satisfied by this,

$$(kY' - Y)'' = 2Y \tag{15}$$

or from the function $Z = Y/k$

$$(k^2 Z')'' = 2Zk \tag{16}$$

The integral representation of Y(k) shows that, of the the three solutions to (15), we are interested in the one that is an odd function of k and that has the following properties when $k \to 0$ and $k \to \infty$:

$$Y(0) = 0,\quad Y'(k) \to \infty \text{ for } k \to 0,$$
$$Y(k) \to 0 \text{ for } k \to \infty.$$

We will not discuss the behavior of Y(k) in detail and derive only the asymptotes for $k \to 0$ and $k \to \infty$. The solutions to (15) for $k \to 0$ can be approximated by $Y_1 \sim k\ln k$, $Y_2 \sim \text{const}$; $Y_3 \sim k$. From (17) we have

$$Y(k)|_{k\to 0} \sim k\ln|k| \sim y\ln\frac{|y|}{\rho_T}. \tag{18}$$

This means that the current density becomes zero at y = 0 and rises somewhat more rapidly than linearly away from the wall.

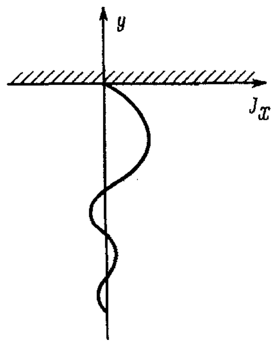

Fig. 2

The solutions to (15) for $k \rightarrow \infty$ can be approximated by

$$Y_{1,2,3} \sim k \exp(|k|^{2/3} a_{1,2,3});$$

$$a_{1,2,3} = \frac{3}{2^{2/3}} \exp \frac{2\pi i n}{3}; \quad n = 1, 2, 3. \tag{19}$$

It follows from (17) that $Y = C_1 Y_2 + C_2 Y_3$, where C_1 and C_2 are certain constants, so

$$Y(k) \sim k \exp\left(-\frac{3}{2^{5/3}} |k|^{2/3}\right) \times$$

$$\times \cos\left(\frac{3\sqrt{3}}{2^{5/3}} |k|^{2/3} + \text{const}\right). \tag{20}$$

Figure 2 shows the general form of Y(k). As would be expected, the current actually is localized within a layer of thickness about ρ_T.

The total current flowing in the layer

$$I_x = \int_{-\infty}^{0} j_x dy \tag{21}$$

and this is (with $c_{T_1} = c_{T_0}$))

$$I = \frac{en_+}{\sqrt{\pi}} \frac{c_{T0}}{\omega_e} c \frac{E}{H} = \frac{en_+}{\sqrt{\pi}} \rho_T c \frac{E}{H} \tag{22}$$

This expression has an obvious physical significance. If we use (22) with the assumption that the noise plays no great part, we can readily derive the voltage-current curve for any particular system.

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